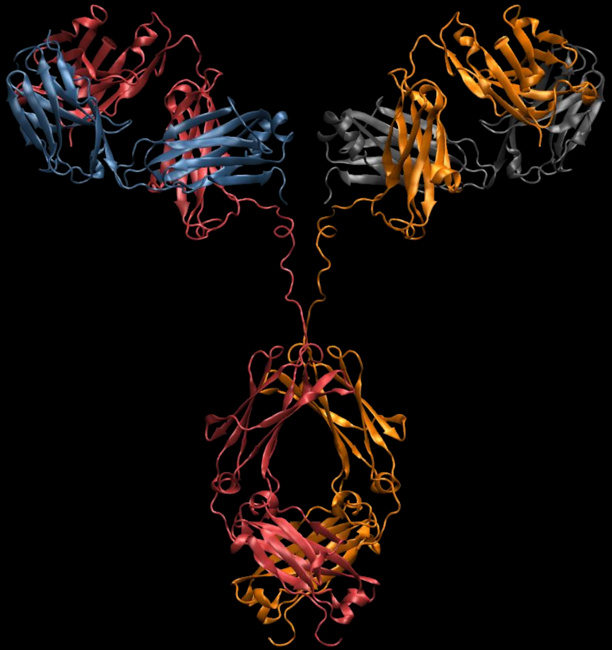
SasCalc
Periodic Boundary Conditions

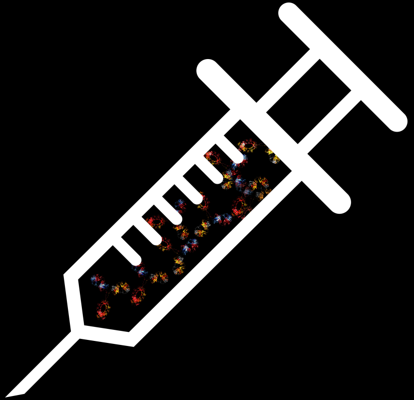
Ian Hunt-Isaak, Joseph E. Curtis, Steven C. Howell

Road Map

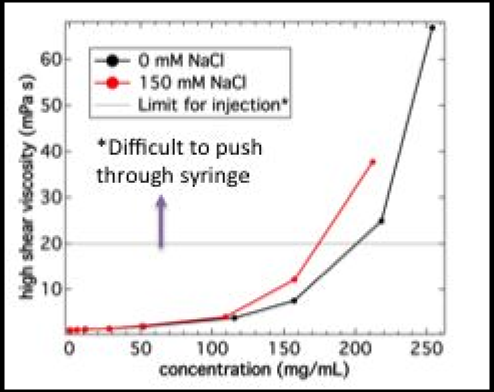
1. Question to answer
2. Issue with current approach
3. Alternative approaches
4. Results
5. Next Steps

Why are concentrated proteins so viscous?

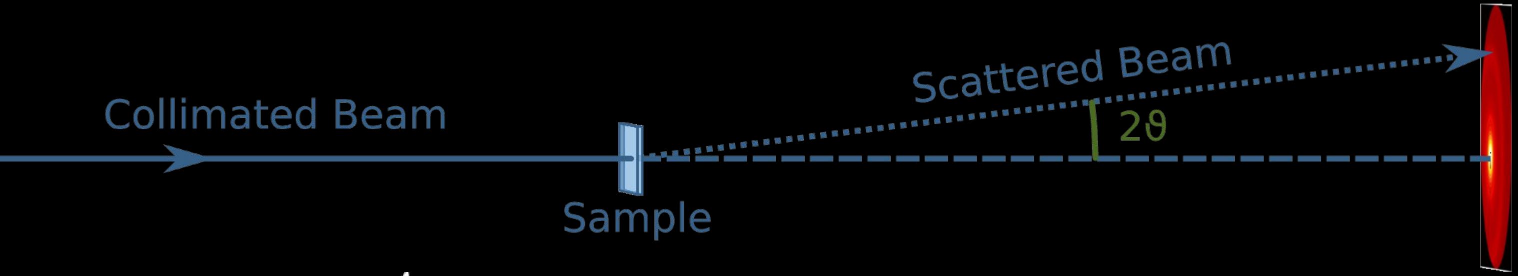


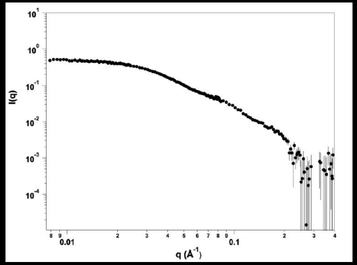


https://openclipart.org/image/2400px/svg_to_png/205886/remigho-syringe.png



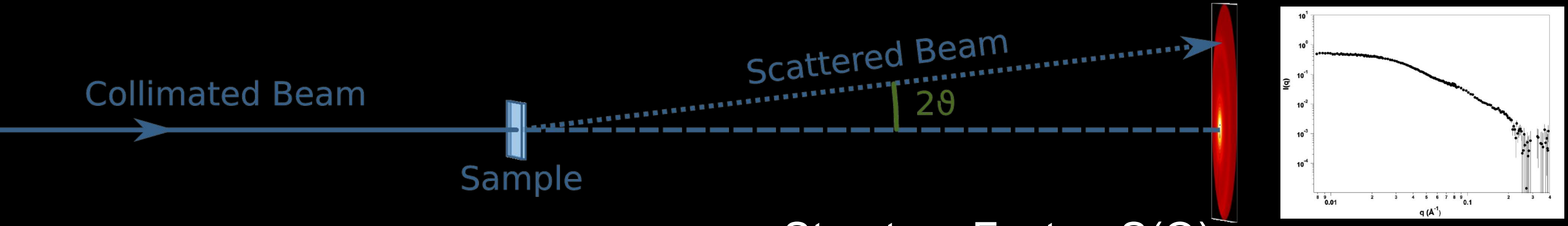
Small-Angle Scattering (SAS)

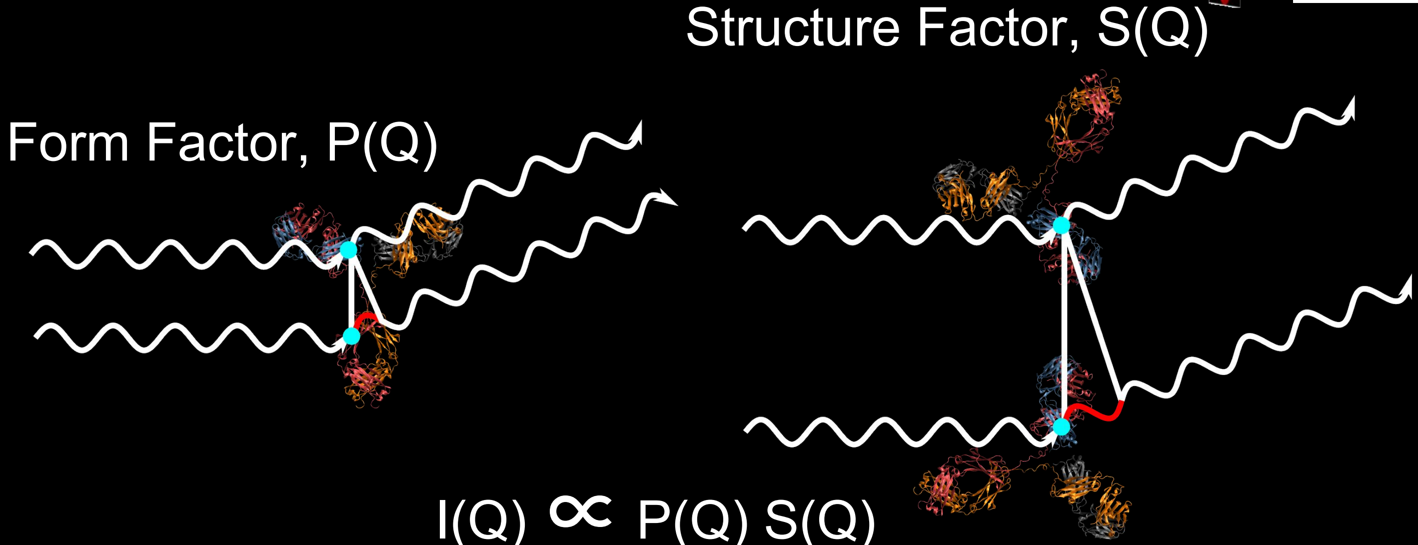




$$Q = \frac{4\pi}{\lambda} \sin(\theta)$$

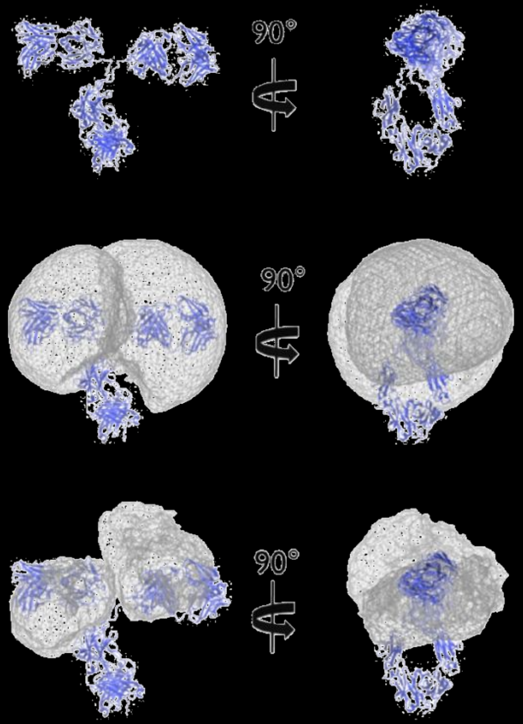
Small-Angle Scattering (SAS)

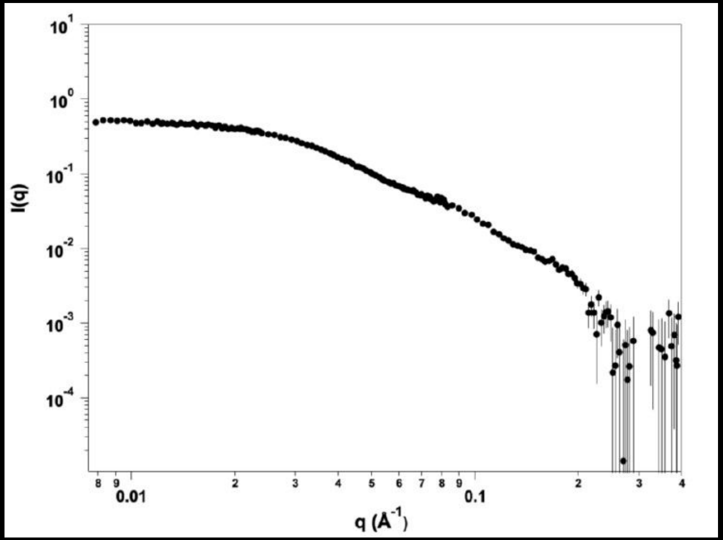




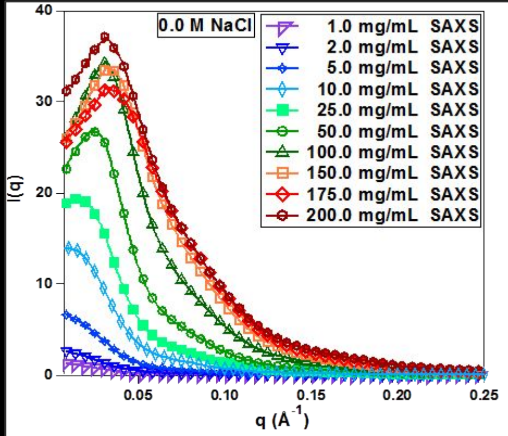
$$I(Q) \propto P(Q)\,S(Q)$$

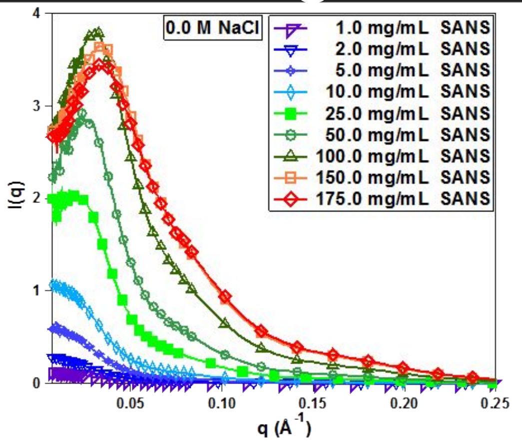
SAS of dilute proteins used to determine shape





Concentration Affects the Scattering





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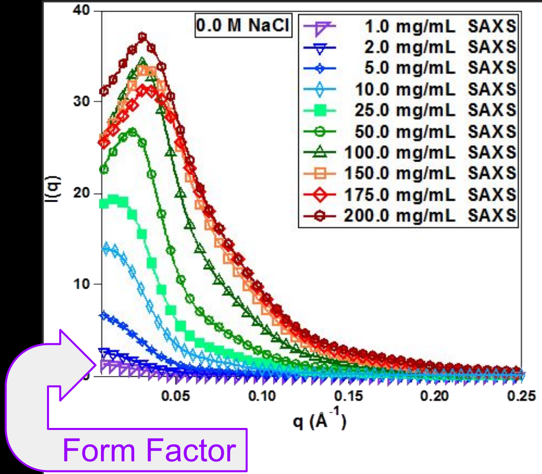
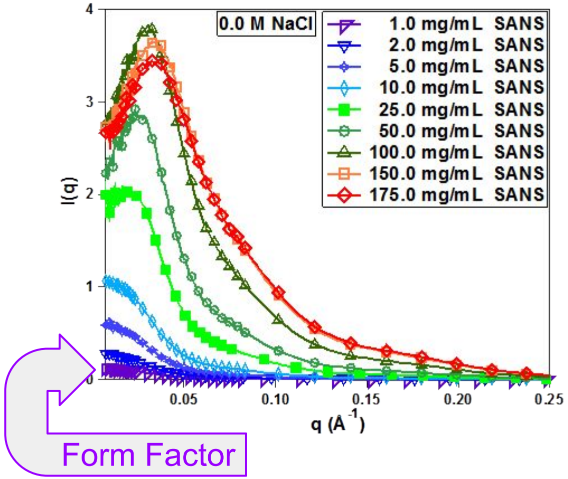
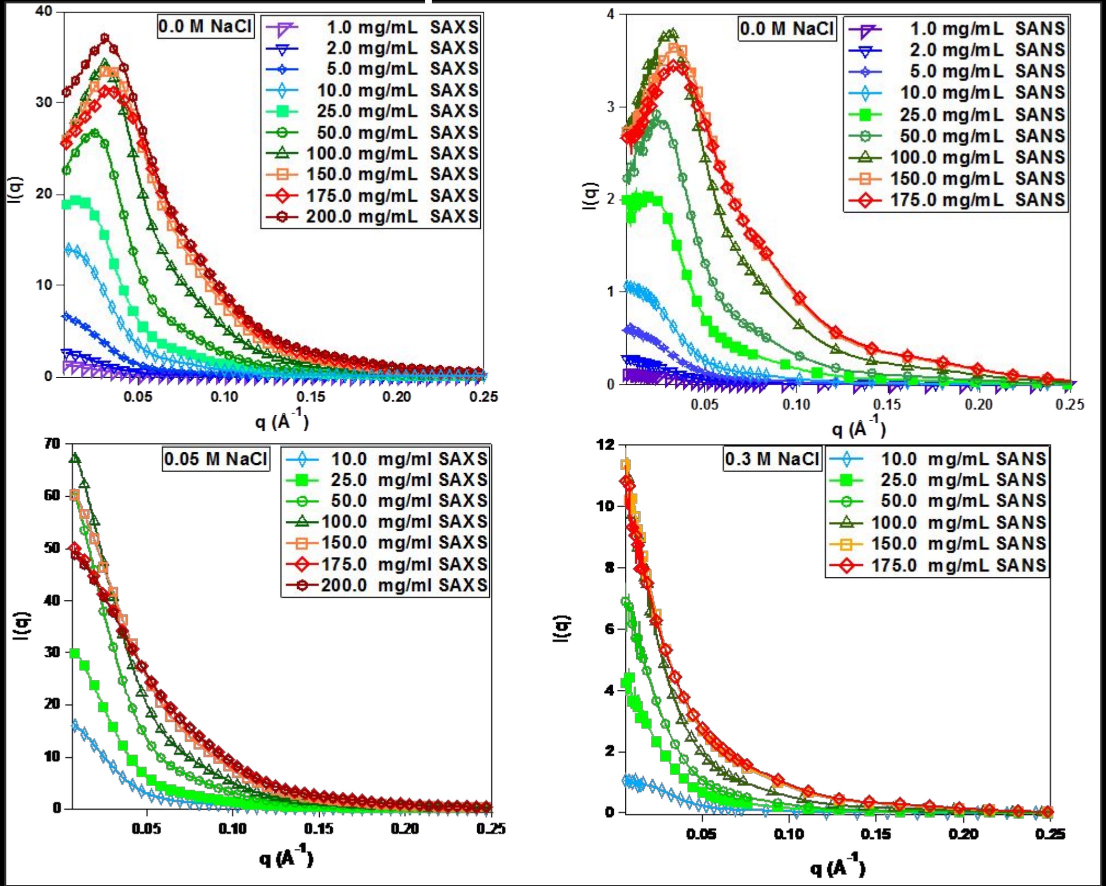


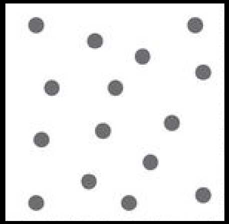


Co-solutes modulate protein interactions



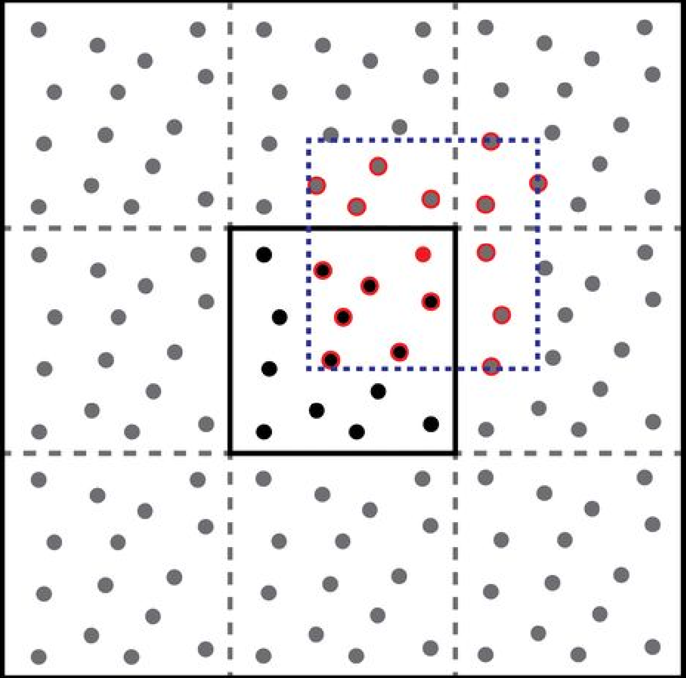
Change in scattering is not dominated by P(q) or S(q) alone.

Molecular Simulation



Adapted from http://nmschneider.com/blog/2014/5/9/nearest-image-for-particle-simulations-in-matlab

Molecular Simulation



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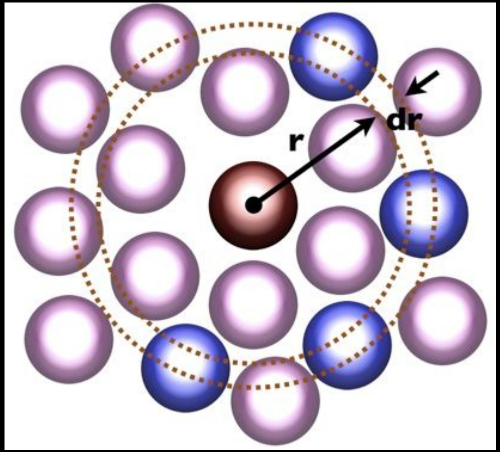
Fourier Transform: g(r) → S(Q)

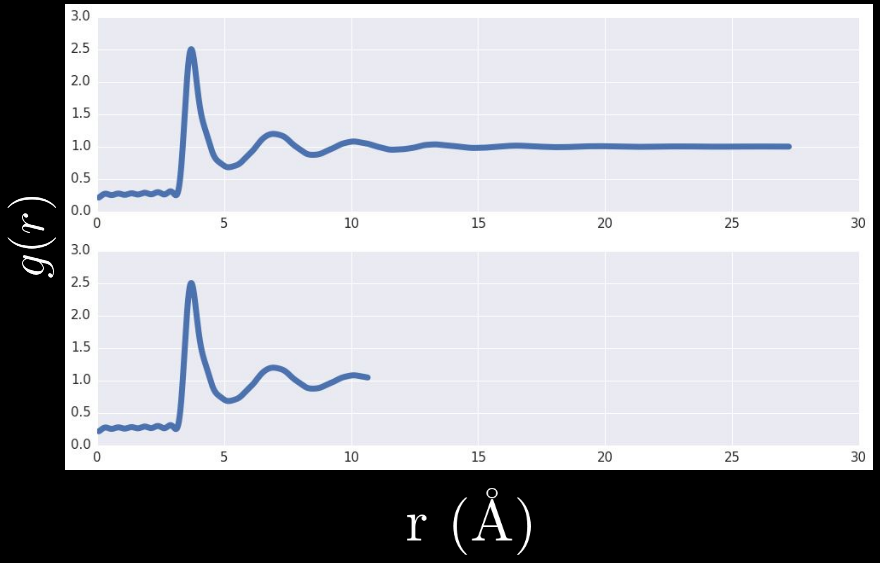


$$S(q) = 1 + 4\pi\rho \int (g(r) - 1) \cdot r^2 \mathrm{sinc}(qr) dr$$

- Limited to spherical systems

Finite size effects

$$S(q) = 1 + 4\pi\rho \int (g(r) - 1) \cdot r^2 \mathrm{sinc}(qr) dr$$



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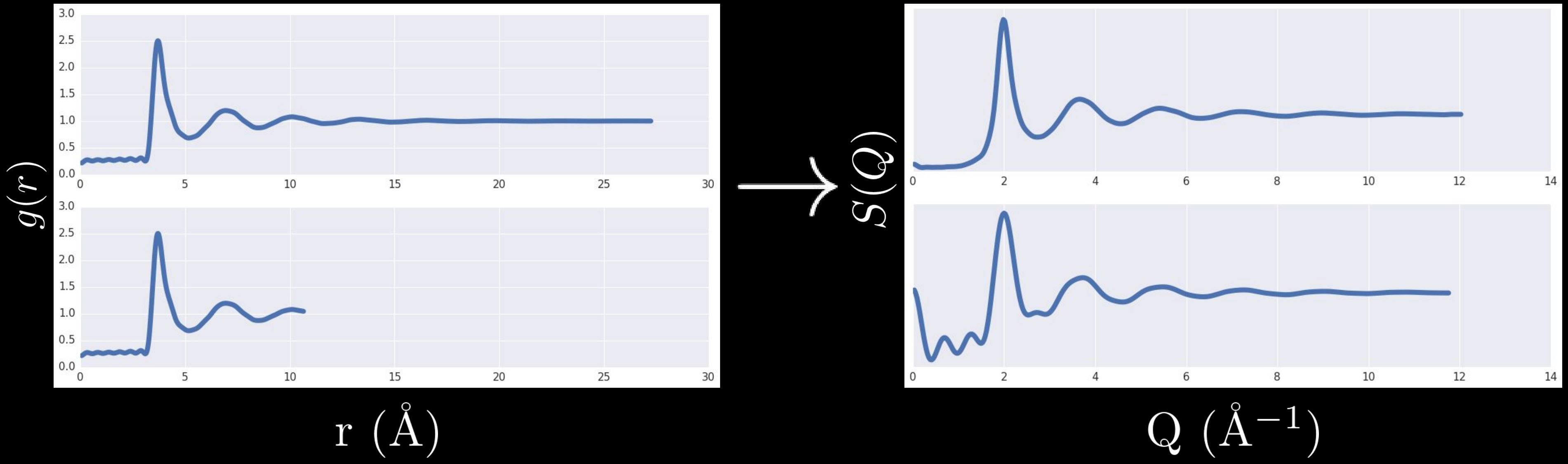


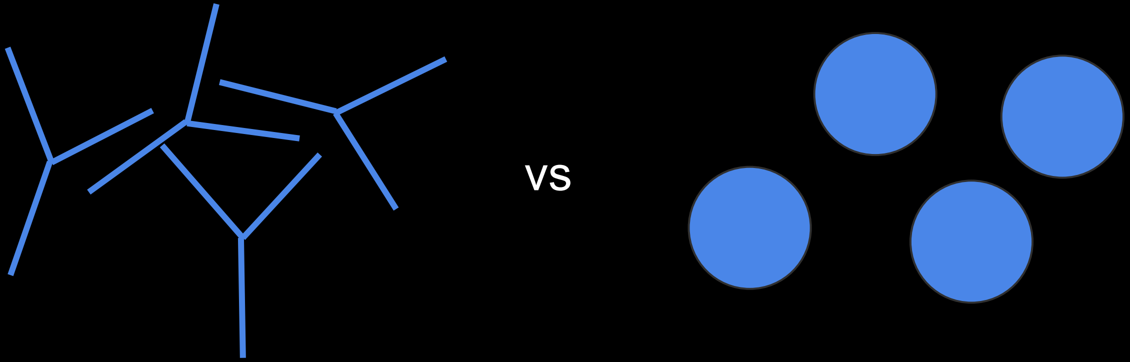
Desired Features of Calculator

- Simultaneously calculate both P(Q) and S(Q)
- Avoid finite size effects
- Fast

Debye Formula

$$I(q) = \sum_{j=1}^{N} \sum_{k=1}^{N} f_i(q) f_j(q) \frac{\sin(qr_{ij})}{qr_{ij}}$$

Not universally applicable:



Explicit Fourier Transform

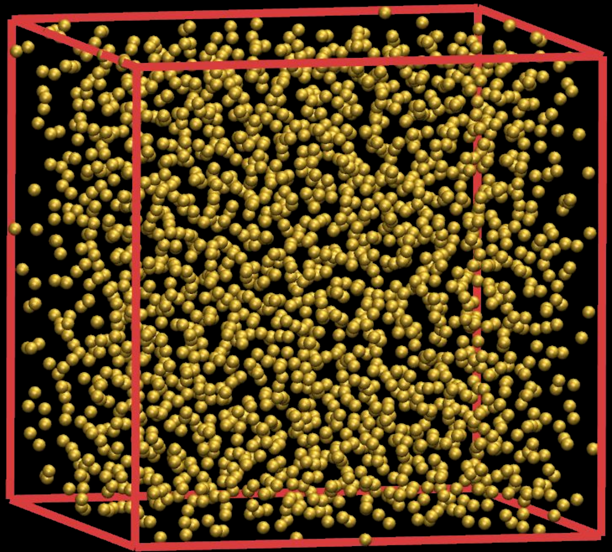
- Most general
  - Contains P(Q) and S(Q)

$$I(q) = I_1(q)\langle \sum_{j=1}^{N} \sum_{k=1}^{N} e^{-i\boldsymbol{q}\boldsymbol{r}_{jk}} \rangle = I_1(q)\langle \sum_{j=1}^{N} \sum_{k=1}^{N} \cos \boldsymbol{q}\boldsymbol{r}_{jk} \rangle$$

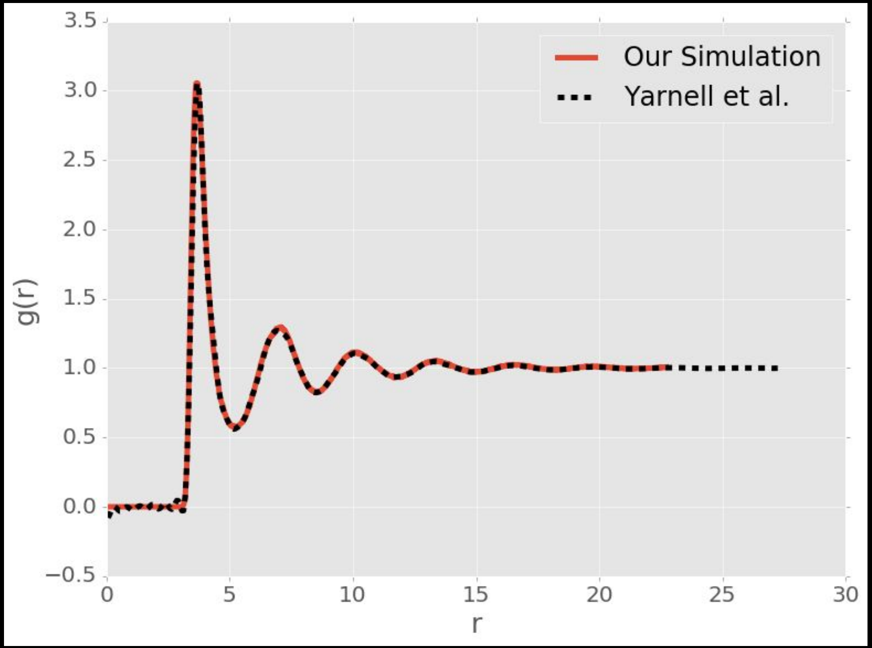
Removing finite size effects

- Fancy Stat Mech - g(r)
  - Limited to spherical systems
- Bigger Box
  - Num Atoms ~ (Box Length)^3
- Calculate Box scattering and remove

First Test System - Lennard Jones Particles

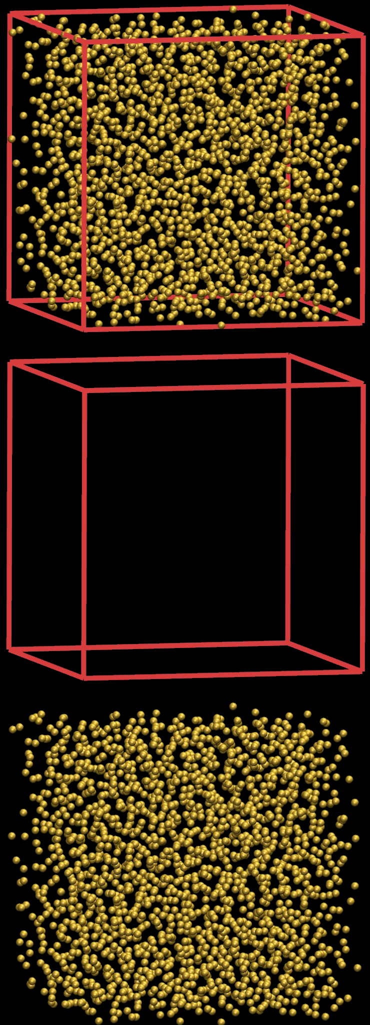


Verify Radial Distribution of Simulation

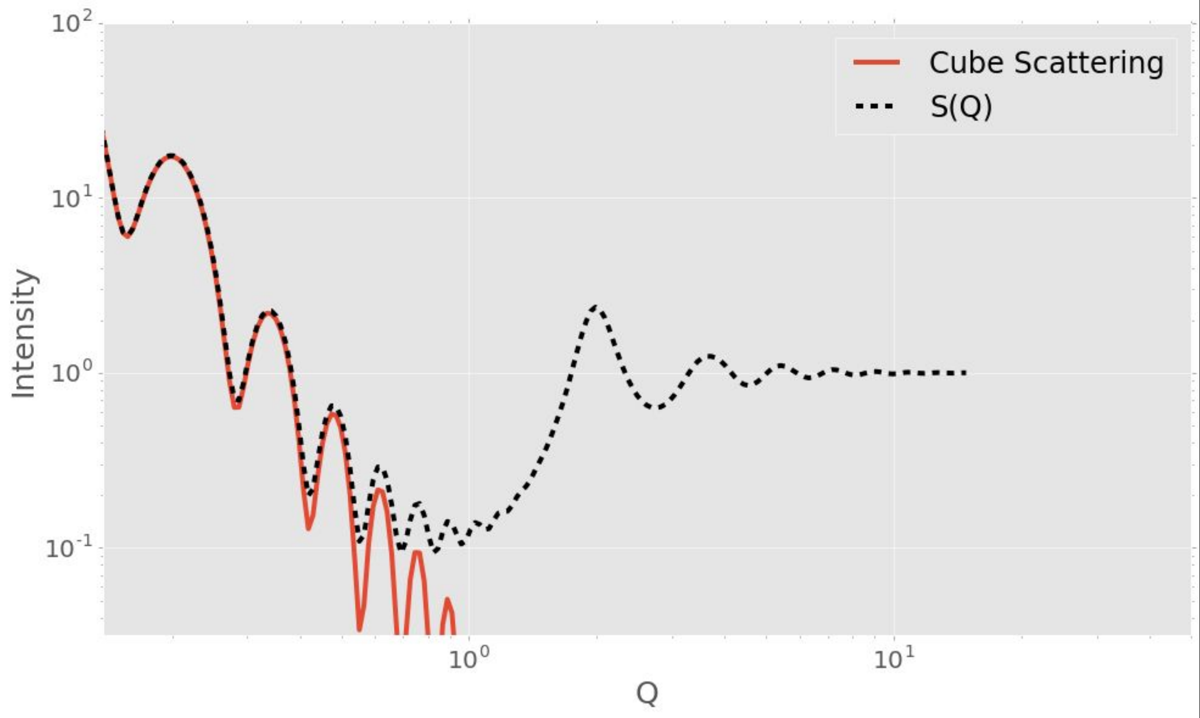


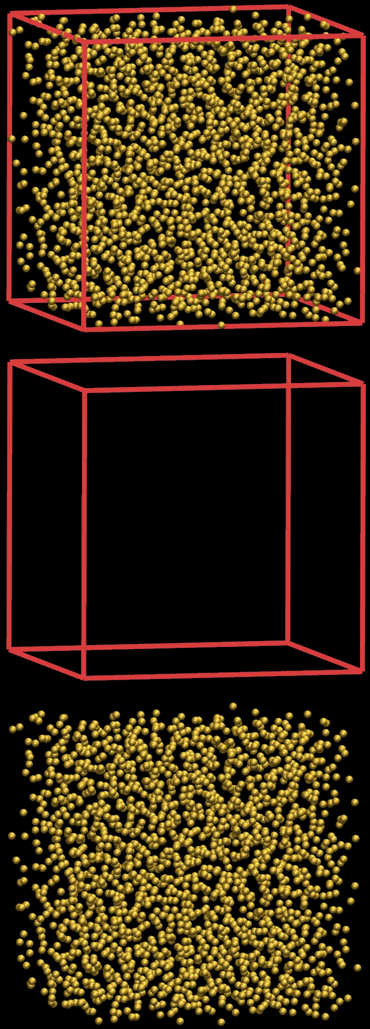
Radial Distribution Function for Liquid Argon at 85 K - Yarnell, et al.

Removing Box Effects



Removing Box Effects





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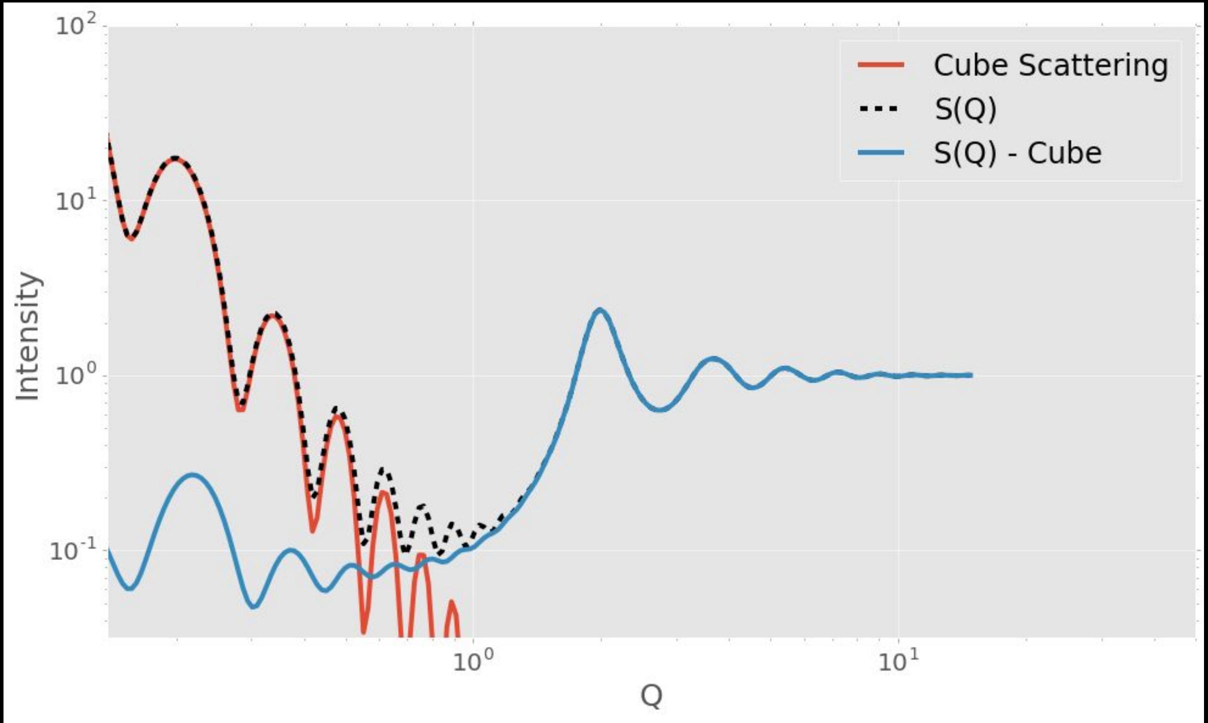
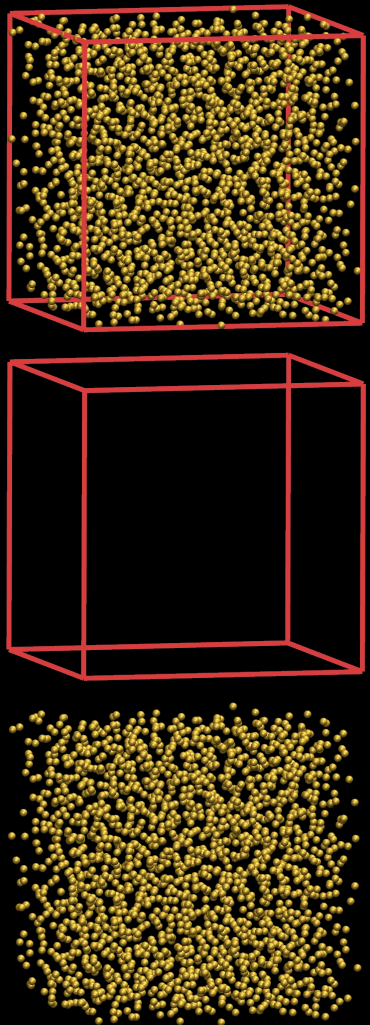
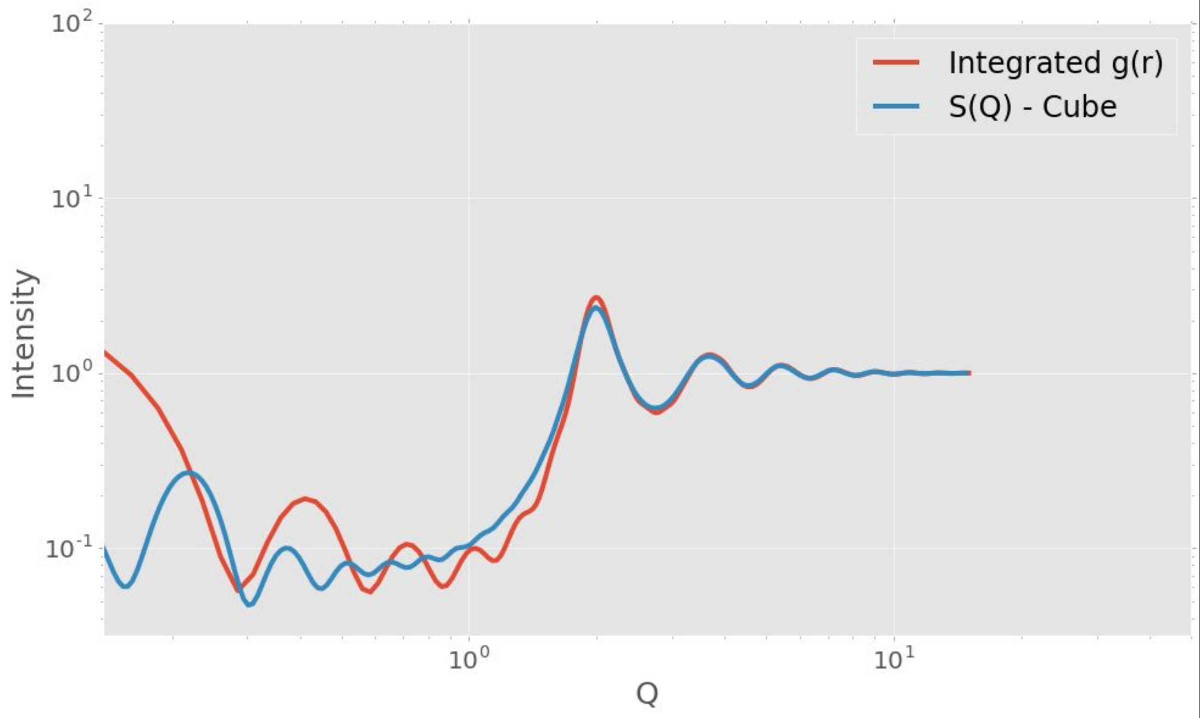
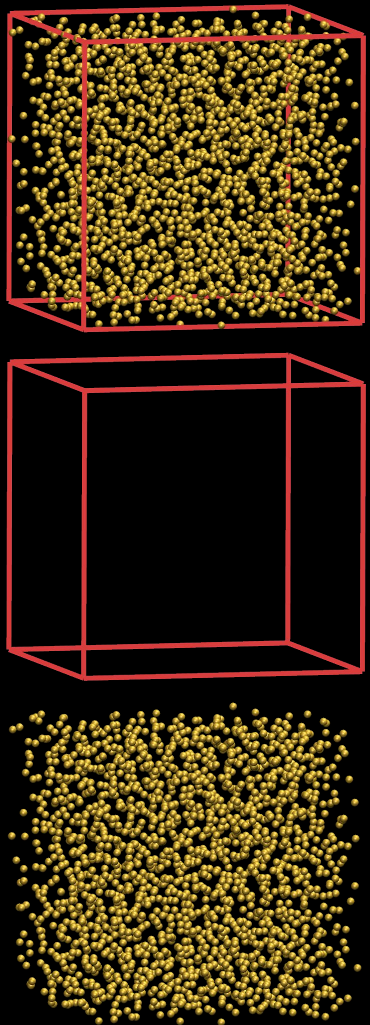




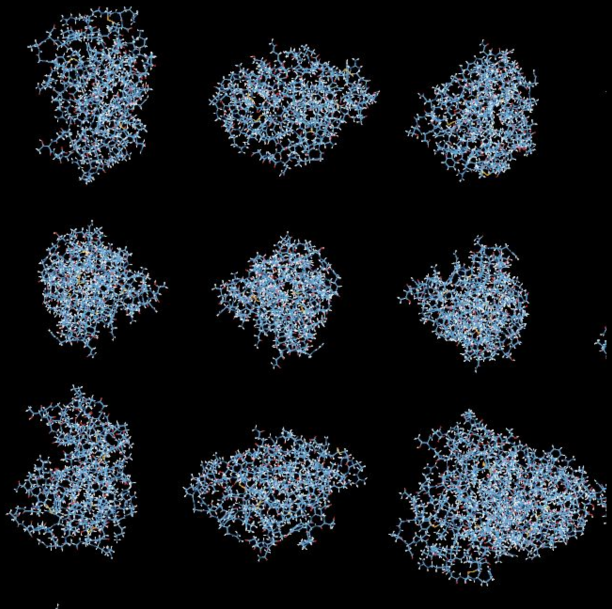
Integrating g(r) vs Box Subtraction

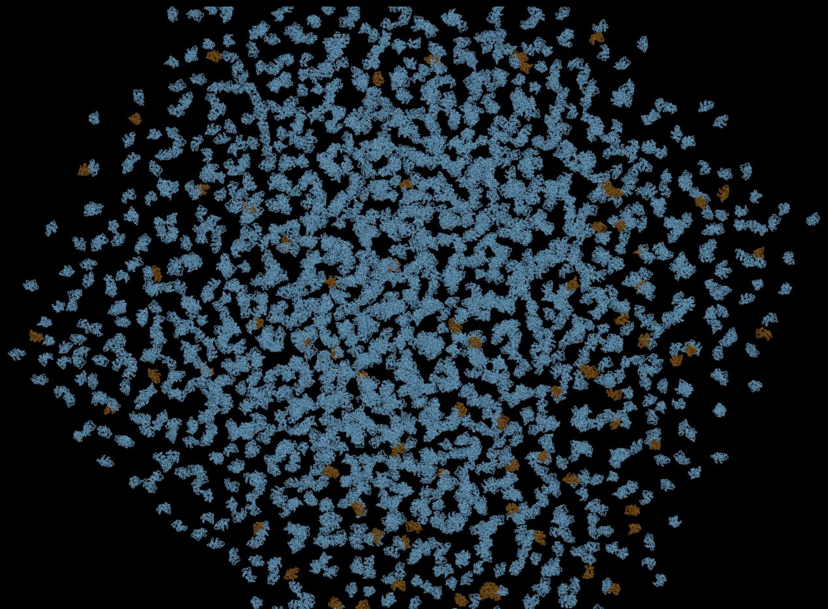




Next Steps

- Apply this to a periodic box of proteins (lysozyme, mAb)
- Automate algorithm to subtract box effects
- Parallelize using GPUs





Thank you!

OBERLIN COLLEGE PHYSICS

NIST

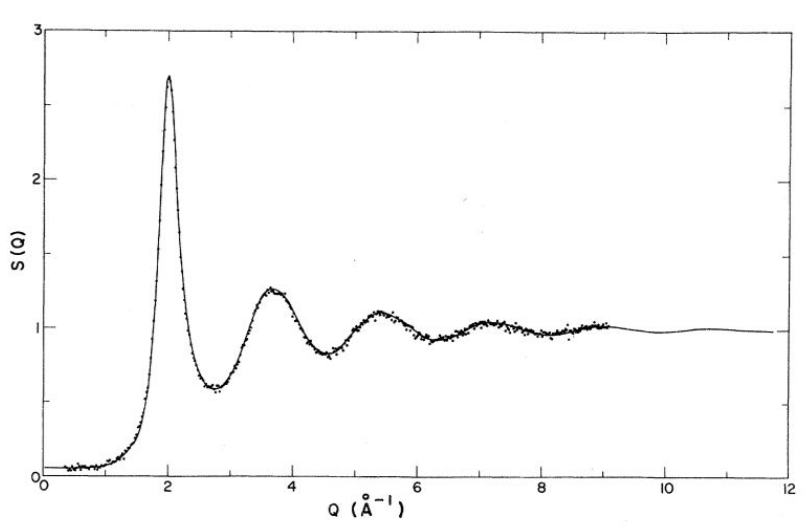
NSF CHRNS CENTER FOR HIGH RESOLUTION NEUTRON SCATTERING NIST

NIST Center for Neutron Research

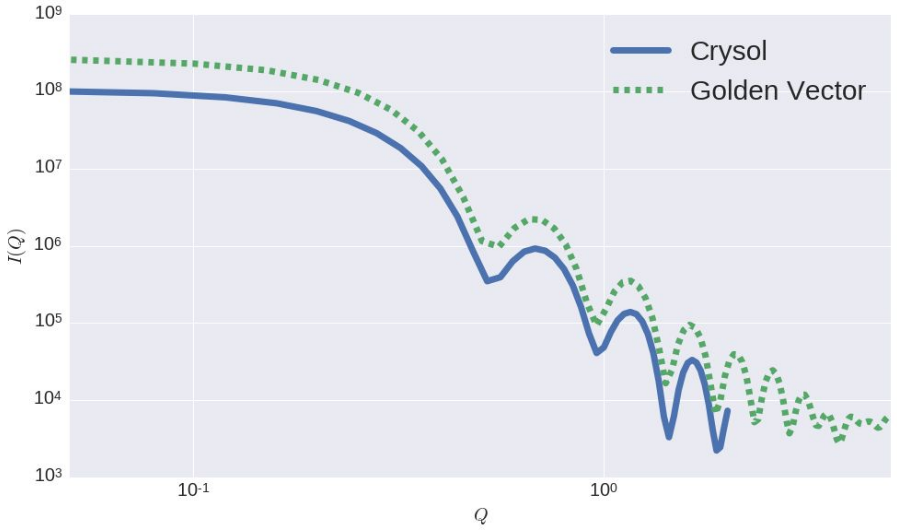
Backup Slides

Existing Calculators

Experiment



Simulation



Watch out S(Q) ≠ I(Q).

S(Q) via Fourier Transform

- g(r) simple to calculate
- Can extend h(r) via use of Ornstein-Zernike Equation

$$h(r) = g(r) - 1$$

$$S(Q) = \frac{1}{1 - \widetilde{h}(q)}$$

SasCalc (Golden Vector)

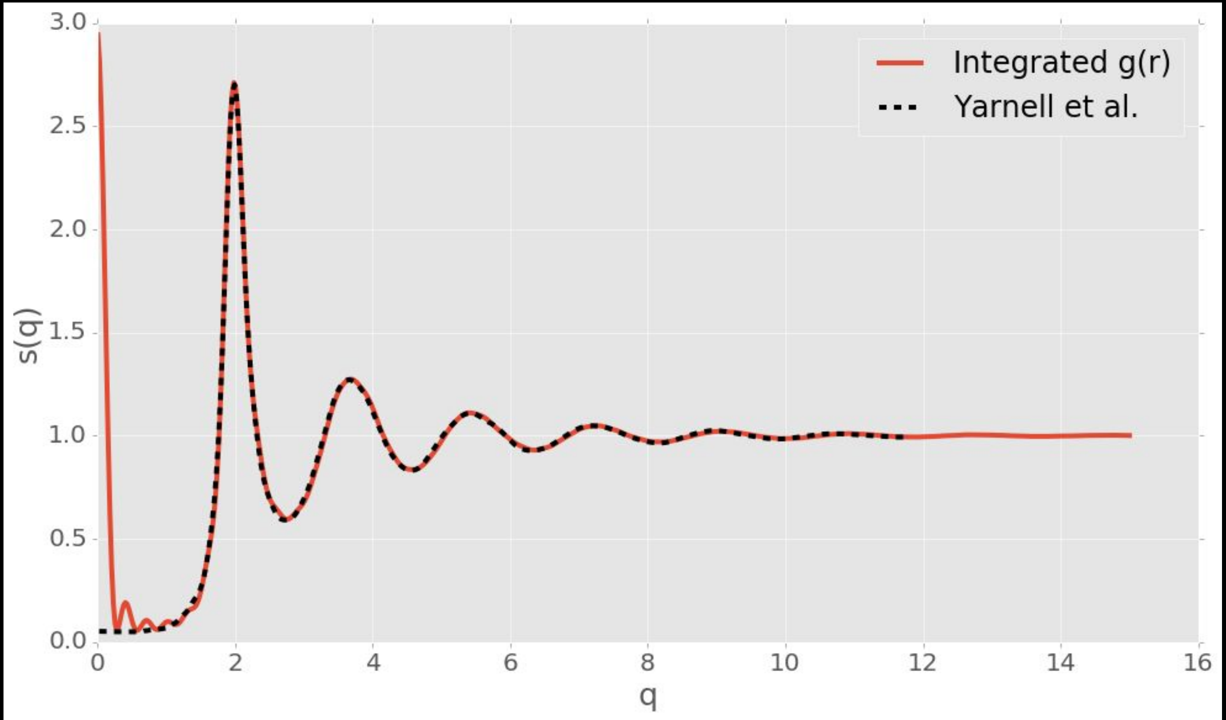
$$I(\mathbf{q}) = \left[\sum_{j}^{N} b_j \cos(\mathbf{q}\cdot\mathbf{r}_j)\right]^2 + \left[\sum_{j}^{N} b_j \sin(\mathbf{q}\cdot\mathbf{r}_j)\right]^2$$

- Multiple Molecules?
- Separation of S and P?

Two questions:

- Can Extract S?
- Periodic Boundary Conditions effects?

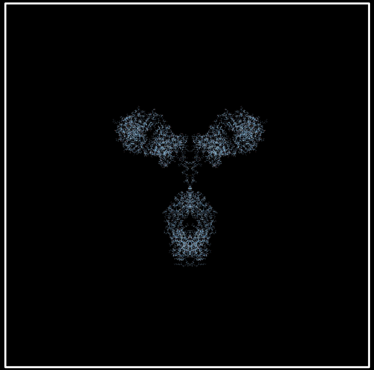
Scattering of test system

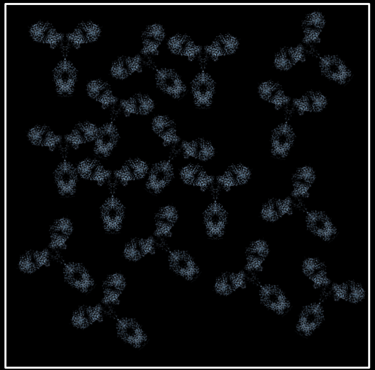


Current Limitations

	10 mg/ml	100 mg/ml
Of Medical interest?	YES	YES
Can we calculate Scattering?	YES	NO (somewhat)

Simulation Boxes:





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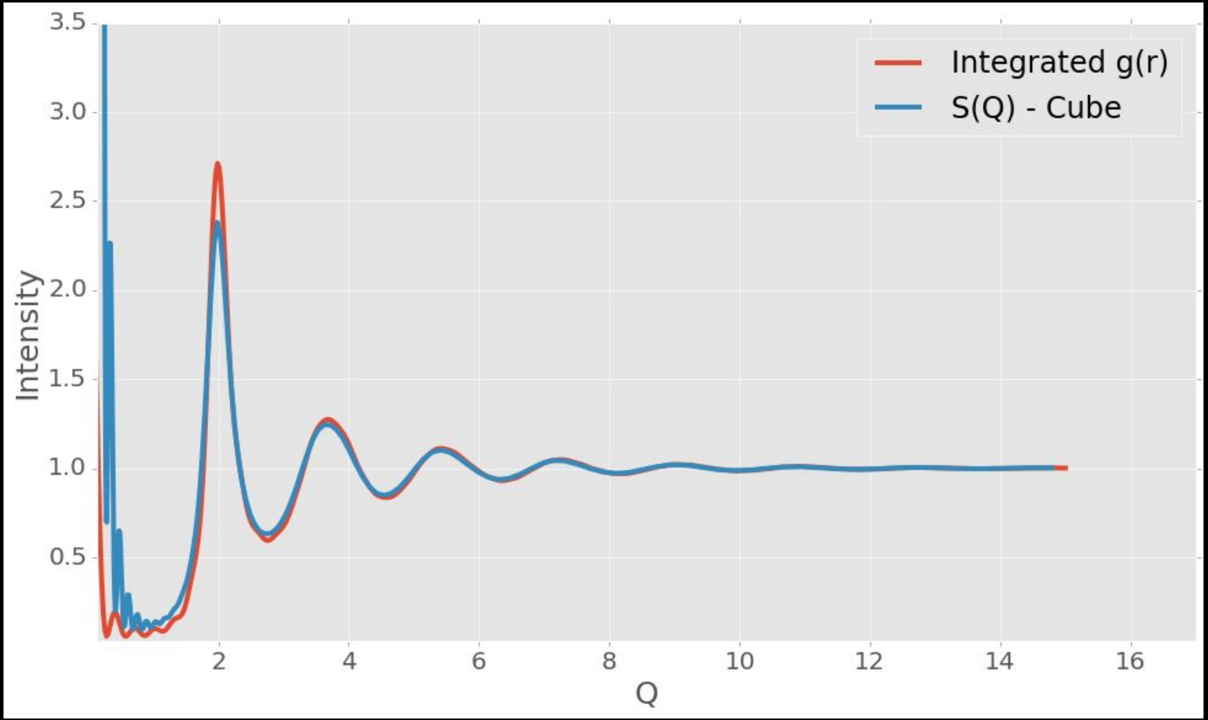

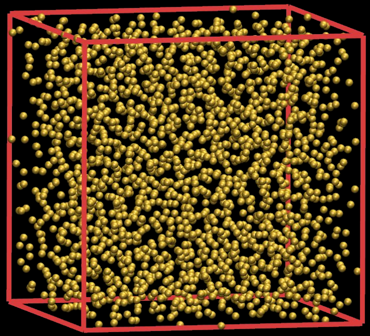

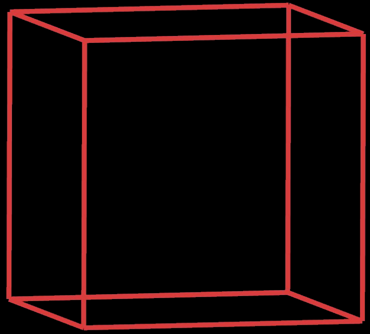

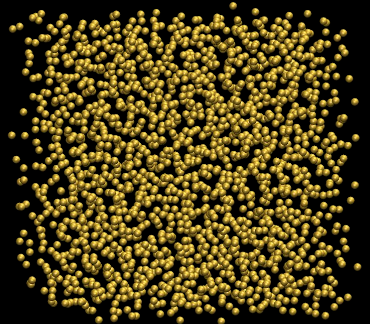

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